Lecture 7: State Space Representation of Log-linearized Model

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The state space representation of a model is very useful to do some subsequent analysis after solution.

1 The State Space Representation

The log-linearized model can be written as the following form:

$$E_t\, X_{t+1} \;=\; \underbrace{M}_{(n+m)\times(n+m)} \cdot \underbrace{X_t}_{(n+m)\times 1}$$

where n, m stand for the number of jumps (control variables) and states/predetermined variables respectively. If we define:

$$X_t \;\equiv\; \begin{pmatrix} \underset{n\times 1}{X_{1t}} \\ \underset{m\times 1}{X_{2t}} \end{pmatrix}$$

and

$$M \;\equiv\; \begin{pmatrix} \underset{n\times n}{M_{11}} & \underset{n\times m}{M_{12}} \\ \underset{m\times n}{M_{21}} & \underset{m\times m}{M_{22}} \end{pmatrix}$$

where M is the coefficient matrix.

After we have the policy function $X_{1t} = \underset{n\times m}{\phi} X_{2t}$, then we have:

$$\begin{aligned} E_t X_{1t+1} &= M_{11} X_{1t} + M_{12} X_{2t} \\ &= \left(M_{11}\phi + M_{12}\right) X_{2t} \\ &\equiv \underset{n\times m}{C}\, X_{2t} \end{aligned} \tag{1}$$

and

$$
\begin{aligned}
E_t X_{2t+1} &= M_{21} X_{1t} + M_{22} X_{2t} \\
&= \left(M_{21}\phi + M_{22}\right) X_{2t} \\
&\equiv \underset{m\times m}{A} \; X_{2t}
\end{aligned}
$$

By rational expectation,

$$X_{2t+1} = AX_{2t} + B\epsilon_{t+1}$$

or

$$X_{2t} = AX_{2t-1} + B\epsilon_t$$

Hence, by the policy function, we have:

$$
\begin{aligned}
X_{1t} &= \phi X_{2t} \\
&= \phi A X_{2t-1} + \phi B \epsilon_t \\
&\equiv C X_{2t-1} + D\epsilon_t
\end{aligned} \tag{2}
$$

In the above equation, we use equivalent notation. You can numerically verify $C = \phi A$ using Matlab and a simple RBC model (see the Matlab code in the next lecture notes about BK method). Hence, the state space represtation is:

$$
\boxed{
\begin{aligned}
X_{1t} &= CX_{2t-1} + D\epsilon_t \qquad (3) \\
X_{2t} &= AX_{2t-1} + B\epsilon_t \qquad (4)
\end{aligned}
}
$$

2 MatLab code

Refer to the next lecture notes about BK method.